# Study Guide for Unit 2—More Derivatives Test

From Unit 1:
Basic Limits
Product Rule
Definition of a Derivative
Position, Velocity, Acceleration
Continuity and Differentiability
Linearization

From Unit 2:
Trig Derivatives
Chain Rule
Squeeze Theorem
Inverse Trig
Implicit Differentiation
Exponential and Logarithm Derivatives
L'hopital

You must know ALL the rules for finding derivatives. (power, quotient, product, chain rule, exponential, logarithmic, trig and inverse trig)
Refer to your packet sheets and book homework – finish these if you haven't yet.
Practice "saying and doing" the power rule and quotient rule.
Make flashcards to memorize all the other derivative rules.

Know the UNIT CIRCLE.
Know how to write the equation of a tangent line and/or a normal line!
When and how to use implicit differentiation.
Velocity is first derivative of position.
Acceleration is first derivative of velocity and second derivative of position.

CONTINUITY: graphically AND in terms of limits. (Be able to justify using the calculus definition of continuity.)
CONTINUITY: a requirement for differentiability but continuity does NOT imply differentiability

Be able to write absolute value functions, especially the parent function, as a piecewise function. Determine continuity and differentiability of a piecewise function.

What do we know about horizontal tangents? Vertical Tangents?
How can the calculator be helpful:
nDeriv for finding the value of the derivative at a specified x value
using the graphing feature of the calculator to solve equations
(ex: where is the tangent of $y=e^x$ parallel to the tangent of $y = x^2$?
What does linearization mean? Review your notes on using a linearization to approximate values on a curve.

## Multiple Choice Practice:

1. $\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 4}$ is

(A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent

2. If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$

(A) 10 (B) 9 (C) 7 (D) 5 (E) 3

3. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$ (B) $y + 3 = -\frac{7}{3}(x - 1)$ (C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$ (E) $y + 3 = \frac{11}{3}(x - 1)$

4. Let $f$ be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of $f$ at $x = 3$?

(A) $-5$ (B) $-\frac{15}{4}$ (C) $-1$ (D) 6 (E) 17

5. Let $f$ be the function given by $f(x) = \frac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of $x$ is $f$ not continuous ?

(A) $-3$ and $-1$ only (B) $-3, -1,$ and 2 (C) $-1$ only

(D) $-1$ and 2 only (E) 2 only

6. For which of the following does $\lim_{x\to\infty} f(x) = 0$?

***I.*** $f(x) = \frac{lnx}{x^{99}}$ ***II.*** $f(x) = \frac{e^x}{lnx}$ ***III.*** $f(x) = \frac{x^{99}}{e^x}$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

7. Let $f$ be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all $x$. Of the following, which is not a possible value for $f(2)$?

(A) $-10$ (B) $-5$ (C) 0 (D) 1 (E) 2

8. Let $f$ be the function given below. What are all values of $a$ and $b$ for which $f$ is differentiable at $x = 1$?

$$f(x) = \begin{cases} x + b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and $b$ is any real number

(D) $a = b + 1$, where $b$ is any real number

(E) There are no such values of $a$ and $b$.

9. The table at right gives values for the functions $f$ and $g$ and their derivatives at $x = 3$. Let $k$ be the function given by $k(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

| $f(3)$ | $g(3)$ | $f'(3)$ | $g'(3)$ |
|---|---|---|---|
| –1 | 2 | 5 | –2 |

(A) $-\frac{5}{2}$  (B) $-2$  (C) 2  (D) 3  (E) 8

10. If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$  (B) $\frac{15}{\sqrt{10}}$  (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$  (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$  (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

11. If $\lim_{h\to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = \sqrt{2}$, which of the following could be the value of $a$ ?

(A) $\frac{\sqrt{2}}{2}$  (B) $\frac{\sqrt{3}}{2}$  (C) $\sqrt{3}$  (D) $\frac{1}{2}$  (E) 2

12. If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

(A) $-2$  (B) $2x + y - 2$  (C) $2x + y$  (D) $4x + 2y - 2$  (E) $y - \frac{y}{x}$

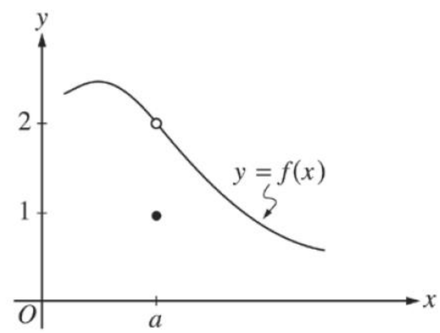
13. Let $f$ be the function given by $f(x) = (x^2 - 2x - 1)e^x$.

(a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

$\lim_{x\to\infty} f(x) =$ ____________

$\lim_{x\to-\infty} f(x) =$ ____________

14. The graph of a function $f$ is shown in the figure below. Which of the following statements is true?




(A) $f(a) = 2$

(B) $f$ is continuous at $x = a$

(C) $\lim_{x\to a} f(x) = 1$

(D) $\lim_{x\to a} f(x) = 2$

(E) $\lim_{x\to a} f(x)$ does not exist

15. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos\sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

16. Write the equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 2$.

Released m/c

1.(2003 AB24 – no calc)

Let *f* be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of *f* at the point where *x* = -1?

A. $y = 7x - 3$ B. $y = 7x + 11$ C. $y = -5x - 5$ D. $y = 7x + 7$ E. $y = -5x - 1$

2.(1998 AB18 – no calc)

An equation of the line tangent to the graph of $f(x) = x + \cos(x)$ at the point $(0,1)$ is

A. $y = 2x + 1$ B. $y = x$ C. $y = 0$ D. $y = x + 1$ E. $y = x - 1$

**Follow Up…Use this tangent line to estimate f(0.1)**

3.(1997 AB10 – no calc)

An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

A. $y - 1 = -\left(x - \frac{\pi}{4}\right)$ B. $y = 2\left(x - \frac{\pi}{4}\right)$ C. $y = -2\left(x - \frac{\pi}{4}\right)$

D. $y - 1 = -2\left(x - \frac{\pi}{4}\right)$ E. $y = -\left(x - \frac{\pi}{4}\right)$

4.(2003 AB89 – calc permitted)

Let *f* be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let *g* be the function $g(x) = x \cdot f(x)$. Which of the following is an equation of the line tangent to the graph of *g* at the point where $x = 2$?

A. $y = 3x$ B. $y - 6 = -5(x - 2)$ C. $y - 6 = -10(x - 2)$

D. $y - 3 = -5(x - 2)$ E. $y - 6 = -7(x - 2)$

5.(1997 AB12 – no calc)

At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line paralel to the line $2x - 4y = 3$?

A. $\left(\frac{1}{2}, -\frac{1}{2}\right)$ B. $\left(\frac{1}{2}, \frac{1}{8}\right)$ C. $\left(1, -\frac{1}{4}\right)$ D. $\left(1, \frac{1}{2}\right)$ E. $(2, 2)$

From: https://1.cdn.edl.io/Uy6aiAbeiTfPsrRQNJjr77OvLkV9Y7ymnoJpXTM8ASNrichD.pdf

27. At $x=3$, the function given by $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x\geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

From: file:///C:/Users/ldoan2/Downloads/201608_Test_2A.pdf

1.
**(3 pts.)** Find the **second** derivative $\frac{d^2y}{dx^2}$ if $x - y^2 = 1$.

**a)** $\frac{d^2y}{dx^2} = -\frac{1}{4y^3}$ **b)** $\frac{d^2y}{dx^2} = -\frac{1}{2y}$ **c)** $\frac{d^2y}{dx^2} = \frac{1}{4y^3}$ **d)** $\frac{d^2y}{dx^2} = \frac{1}{2y}$

5.
**(3 pts.)** Find the derivative of $y = \frac{5 + 5\cot^2 x}{3\csc x}$. Hint: Simplify before finding the derivative.

**a)** $y' = -\frac{5}{3}\csc x \cot x$ **b)** $y' = -\frac{1}{3}\csc x \cot x$

**c)** $y' = \frac{-10\csc^2 x \cot x}{3\sec x \tan x}$ **d)** $y' = \frac{5}{3}\sec x \tan x$

8.
**(3 pts.)** Evaluate $\lim_{x\to 0}\frac{3\sin(ax)}{bx}$ ( constants $a$ and $b$, where $b \neq 0$ )

**a)** $\frac{3}{b}$ **b)** $\frac{3a}{b}$ **c)** $\frac{3}{ab}$ **d)** $\frac{3}{a}$

2. **(12 pts.)** A feather is dropped on the moon from a height of 40 meters. Its height $s$ (in meters) above the surface of the moon $t$ seconds after it is dropped is given by $s(t) = 40 - 0.8t^2$.

a) **(4 pts.)** At what time will the feather be 20 meters above the surface?

b) **(6 pts.)** Find the velocity of the feather the moment it strikes the surface.

c) **(2 pts.)** What is the acceleration due to gravity on the moon?

b. **(5 pts.)** Find $g'(x)$ if $g(x) = \ln\left(\frac{5+\sin x}{7^x}\right)$

9. **(3 pts.)** Find the equation of the line tangent to the graph of $y = 2x - \cos(2x)$ at $x = \frac{\pi}{2}$.

**a)** $y = 2x + 1$ **b)** $y = -x$

**c)** $y = -x + 2\pi$ **d)** $y = -x + \pi$

This one is AWESOME!!!! The figure at right shows the graph of the function $g$ and the line tangent to the graph of $g$ at $x = -1$.

Let $h$ be the function given by $h(x) = e^x \cdot g(x)$.

What is the value of $h'(-1)$?

(A) $\frac{9}{e}$ (B) $\frac{-3}{e}$ (C) $\frac{-6}{e}$ (D) $\frac{-6}{e} - \frac{3}{e^2}$

(E) – 6